Power of Black Hole Physics:
Seeing through the Vacuum Landscape

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Abstract

In this paper we generalize the black hole bound of [1] to de Sitter spaces, and apply it to various vacua in the landscape, with a special emphasis on slow-roll inflationary vacua. Non-trivial constraints on the lifetime and the Hubble expansion rate emerge. For example, the general tendency is, that for the fixed number and the increasing mass of the species, vacua must become more curved and more unstable, either classically or quantum mechanically. We also discuss the constraints on the lifetime of vacua in the landscape, due to decay into the neighboring states.

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1 Introduction

It is usually assumed that from the knowledge of low-energy perturbative physics (e.g., such as, the particle spectrum, and their couplings) in our vacuum, one cannot draw any conclusion about the physics in other vacua on the landscape, without knowing the non-perturbative structure of underlying high scale theory. This belief is based on the intuition, that different vacua correspond to different non-perturbative solutions of the high energy theory, largely separated by the expectation values of the classical order parameters (e.g., vacuum expectation values (VEVs) of the scalar fields), whereas low energy perturbative physics only accounts for small fluctuations about this solutions. As a result, even in the neighboring vacua, physics may be arbitrarily different and unpredictable for a low energy observer in our vacuum. We wish to show that black hole (BH) physics can provide a powerful guideline for overcoming this obstacle. Among, the expected enormity of the vacuum landscape, there is a large subset that shares common gravitational physics. In these vacua, the classical black hole physics is also common and imposes the same consistency constraints on perturbative particle physics.

In particular, by incorporating the consistency bounds, that BH physics imposes on number and masses of particle species [1], we can derive non-trivial constraints not only on our vacuum, but on any quasi-stationary state, which can be obtained by a continuous deformation of it. Under *continuous deformation*, we mean a change of expectation values that preserves invariant characteristics of the vacuum (such as, the number of species, their chirality, and possibly other topological characteristics). In a certain well-defined sense, to be made precise below, BH physics allows us to "see" through the landscape. In this part of the discussion, the key tool in our consideration will be a BH constraint on number of particle species and their masses. This bound can be derived from the flat space thought experiment, with BH formation and evaporation. In this experiment, an observer forms a classical BH and later detects its evaporation products. In each case, when the lifetime of a BH is less than the lifetime of the species, a powerful bound follows. For example, in the simplest case the number of stable species of mass M cannot exceed

$$N_{max} \equiv \frac{M_P^2}{M^2}.$$

(1.1)

This consistency constraint must be satisfied in every vacuum of the theory. This fact automatically limits the number of possible deformations of our vacuum, which from perturbative physics alone one would never guess. For example, in our vacuum, *a priory*, we may have a very large number of massless species coupled to a modulus ϕ. Naively,

nothing forbids existence of another vacuum, obtained by giving an arbitrary VEV to the modulus ϕ. However, since such a deformation of the vacuum gives masses to the species coupled to ϕ, only deformations permitted by the BH bound are possible. Thus, BH physics, automatically constraints physics in such vacua. The vacua in question does not have to be degenerate with ours, or even be stationary. Below we shall generalize BH bound for such vacua. Primary target of this study will be the de Sitter and quasi de Sitter vacua, that may be connected to ours by a continuous deformation of some scalar VEVs. The phenomenological importance of this study is obvious. Existence of such vacua is suggested by the strong cosmological evidence that our Universe underwent a period of inflation, which is responsible for solving the flatness and the horizon problems, and creating the spectrum of density perturbations. Knowing that we, most likely, rolled down from another vacuum, we wish to understand constraints on such states by using BH physics, and whatever knowledge of perturbative physics we have in our present vacuum. The bounds from BH physics, which we discuss in this paper, set powerful criteria about what is the class of effective string actions, which can be consistently coupled to quantum gravity, and eventually capture string physics, which might have been lost in the effective action approach. Those effective field theories or vacua which cannot fulfill this criterion are called swampland [2] (see also [3]).

Our generalization of the BH bound of [1] to the de Sitter and quasi de Sitter vacua relies on certain relations between the Schwarzschild radius and the lifetime of a "test" BH, and the Hubble radius and the lifetime of the corresponding (quasi) de Sitter vacuum respectively. Shortly, for a given number and masses of species, there is an upper limit on the lifetime and the Hubble size of the vacuum, or else the BH bound (1.1) must be satisfied. In the other words, a given vacuum can only invalidate this BH bound on species, by becoming more curved and/or shorter lived. For the slow-roll inflationary vacua, this implies constraints on the slow-roll parameters, and subsequently, on the allowed number of the inflationary e-foldings.

For the classically-stable vacua, the story is a bit more subtle. Naively, since such vacua are exponentially long lived, their lifetime should exceed the lifetime of any sensible BH that can fit within their de Sitter horizon. Hence, such vacua should automatically fall within the validity of our arguments, and the only resulting constraint should be on their curvature scale. However, in the light of enormity of the string landscape, the tunneling rate can be enhanced by number of the neighboring vacua, to which they can decay via quantum-mechanical tunneling. Intuitively it is clear, that at least in weakly-coupled theories, the exponential longevity of meta-stable vacua should be maintained, or else

the vacuum in question can no longer be treated as a well defined quasi-classical state. However, for our purposes in many cases the issue may be quantitative and we therefore perform a brief investigation of this question in the first part of the paper.

Thus, in the first part of the paper we consider constraints on the lifetime of vacua. The purpose of this study is twofold. First, as explained above, we wish to make sure that enormity of the possible decay channels does not interfere with generalization of the BH bound to such meta-stable vacua. Secondly, there is a phenomenological byproduct. Demanding that our vacuum with cosmological constant Λ has a long enough life-time, one gets some constraints on Λ which are related to the number N_{vac} of vacua. As a result we will see that an upper bound on Λ can be obtained, which decreases with the number N_{vac} of vacua (or conversely an upper limit on N_{vac} arises, which decreases with Λ). So, if there exist too many vacua, the cosmological constant must be smaller than its observed value today. As expected, for our present vacuum, these bounds are rather mild, and become only phenomenologically interesting for very large numbers of N_{vac}.

In string theory, vacuum decay processes are related to domain wall configurations. These domain walls can be thought to be built by intersecting (D)-branes of the underlying superstring theory. We will discuss some aspects of vacuum decays in the string landscape, in particular also the possibility of tunneling from de Sitter or Minkowski vacua to anti-de Sitter vacua, which typically arise in flux compactifications. The corresponding domain wall solutions are given by branes that precisely act as sources for the background fluxes. In this way we can derive some constraints on the flux quantum numbers, by requiring Minkowski or de Sitter vacua with long enough life-time.

In section 3 we discuss bounds on the landscape of effective field theories from BH decays. As it was discussed in [1] and also in [4] these bounds provide a possible explanation of various hierarchies observed in nature. Namely in [1] some perturbative and non-perturbative arguments were given that in a quantum field theory with N species of particles of mass M, there is for large N an inevitable hierarchy between the Planck mass M_P and M:

$$M_P^2 \geq N M^2 \,. \tag{1.2}$$

In particular, for N of order $N \sim 10^{32}$, the bound (1.2) explains the hierarchy between M_P and the N species roughly at the TeV-scale. E.g. this large number of particles is realized in scenarios with large number of extra dimensions and 10^{32} KK modes at the TeV-scale [5]. Recently [6], it was argued, that large number of standard-model-like species also leads to the smallness of strong CP parameter.

We will generalize the BH bounds to the case of non-static de Sitter Universes and to quasi de Sitter type time-dependent backgrounds. This will give us new restrictions on inflationary scenarios like chaotic inflation or D-brane inflation. We also consider bounds from BH on landscape models with softly broken supersymmetry in static Universe. We shall see, that the large number and the mass of the species tends to make the vacuum more highly curved and shorter lived.

Finally, we wish to stress an important point concerning the possible relevance of the lowered cutoff of the theory for the generalized BH bound. As it was shown in [4], with increasing number of species, not only their masses, but also the gravitational cutoff of the theory gets lowered and is bounded from above by $M_P/\sqrt{N}$. In particular, this conclusion agrees with the perturbative argument [7, 8] about the one-loop renormalization of the Planck mass by N species. In our constraint of the de Sitter vacua, the central role is played by the bound (1.2), which has to be satisfied by all the relevant (long enough lived species that can fit within the appropriate BH horizon (see below)) species, irrespectively whether they are above of below the cutoff. This fact is important for our applications to the string landscape, as it allows us to constrain vacua in which the masses of the species are way above the string scale, although the latter is the cutoff of the theory. For instance, such are the brane-inflationary vacua in which the heavy species correspond to the lowest excitations of the stretched strings. Such states, although they are heavier than the string scale, fit within the sub-horizon BH, and therefore fall within the validity of the bound [1].

2 Constraints on the life-time of vacua

In this section we provide a discussion about possible restrictions on the maximal number of vacua resp. on the cosmological constant, considering two kinds of decay processes. First we will consider transition between vacua with positive cosmological constant via the creation of expanding bubble. These bubbles are created by Coleman/De Luccia gravitational effects [9] resp. by Hawking/Moss instantons [10]. Then we will discuss the string landscape which typically also contains a large number of AdS vacua with negative cosmological constant. We consider domain wall solutions (membranes in four dimensions) which in analogy to the Coleman/De Luccia instantons can create bubbles of contracting universes, and hence can be responsible for the decay of a Minkowski or de Sitter vacuum into a vacuum with negative cosmological constant.

2.1 De Sitter vacuum decay by quantum tunneling

2.1.1 Creation of a single bubble

It is known from the work of Coleman and De Luccia [9] that a de Sitter universe with cosmological constant $\Lambda \equiv V_0$ can decay into another vacuum de Sitter vacuum or into a Minkowski vacuum which are separated from each other by a potential barrier of height V_1. In Euclidean quantum gravity, the de Sitter entropy of an expanding universe with vacuum energy V_0 is determined by the value of the classical action:

$$S_0 = \frac{24\pi^2}{V_0} . \tag{2.1}$$

Using this value for the Euclidean action, one can compute the 1-instanton decay rate of this De Sitter universe into another vacuum via quantum tunneling in a semiclassical approximation[3]:

$$\Gamma_{(1)} \simeq M_P \exp\left(-\frac{24\pi^2 M_P^4}{V_0} + \frac{24\pi^2 M_P^4}{V_1}\right) . \tag{2.2}$$

Assuming that the height of the barrier is much bigger that Λ, $V_1 >> V_0$, one simply obtains for the life time τ resp. the decay rate Γ:

$$\tau^{-1} \sim \Gamma_{(1)} \simeq M_P \exp\left(-\frac{24\pi^2 M_P^4}{\Lambda}\right) . \tag{2.3}$$

Note that it is also possible that a quantum jump from V_0 to the top of the barrier V_1, which is followed by an decay to another de Sitter vacuum with cosmological constant V_2, where $V_2 > V_0$. This was discussed by Hawking and Moss and is also closely related to thermodynamic fluctuations due to the de Sitter entropy of the vacuum with cosmological constant V_0. Specifically, the decay rate of our vacuum by creation of a single new bubble is given by:

$$\tau^{-1} \sim \tilde{\Gamma}_{(1)} \simeq M_P e^{-\frac{E}{T_H}} , \tag{2.4}$$

where E is the energy necessary to thermally create the new bubble, and T_H is the Hawking temperature of our de Sitter universe:

$$T_H \simeq \frac{\sqrt{V_0}}{M_P} = \frac{\sqrt{\Lambda}}{M_P} . \tag{2.5}$$

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[3]This amplitude was also used in [11] to show that a *single* KKLT vacuum in type IIB superstrings has a life-time longer than the age of the universe.

2.1.2 Decay into N_{vac} vacua

Now we want to consider a much bigger landscape of N_{vac} different vacua, into which our universe can decay via quantum tunneling. First, we consider the case, where all different vacua can be reached by a single tunneling process. Adding up all these 1-instanton decays into the N_{vac} different vacua one simply obtains the following decay amplitude:

$$\Gamma_{(N_{vac})} \simeq N_{vac} M_P \exp\left(-\frac{24\pi^2 M_P^4}{\Lambda}\right). \tag{2.6}$$

Now requiring that for our universe this decay amplitude is suppressed such that our universe has a long enough life-time, i.e.

$$\Gamma_{(N_{vac})} < H = \frac{\sqrt{\Lambda}}{M_P}, \tag{2.7}$$

we derive the following bound on Λ:

$$\Lambda < \frac{24\pi^2 M_P^4}{\ln N_{vac}}. \tag{2.8}$$

E.g. for $N_{vac} = 10^{500}$ one gets $\Lambda/(24\pi^2 M_P^4) < 8.7 \times 10^{-4}$, whereas as for $N_{vac} = 10^{1500}$ one gets $\Lambda/(24\pi^2 M_P^4) < 2.9 \times 10^{-4}$. Again, the obtained bounds on Λ are not very exciting, unless the landscape is extremely huge.

However we should consider not only the 1-instanton process, but also all k-instanton processes, which describe the process that we can reach a certain bubble via the subsequent decay over k different bubbles. In a kind of instanton dilute gas approximation one gets for each step a suppression factor of e^{-S_0}, and hence the decay amplitude for reaching one specific vacuum via k tunneling processes (k-instanton process) becomes

$$\Gamma^k \simeq M_P \exp\left(-\frac{24\pi^2 k M_P^4}{\Lambda}\right). \tag{2.9}$$

In order to obtain the full decay amplitude into N_{vac} different vacua we sum over all possible k-instanton processes, i.e. taking into account all possible ways decay processes. Then we finally obtain:

$$\begin{aligned}
\Gamma^{\text{total}}_{(N_{vac})} &\simeq M_P \left(\sum_{k=0}^{N_{vac}} \frac{N_{vac}(N_{vac}-1)\ldots(N_{vac}-k)}{k!} \exp\left(-\frac{24\pi^2 k M_P^4}{\Lambda}\right) - 1\right) \\
&= M_P \left(1 + e^{-\frac{24\pi^2 M_P^4}{\Lambda}}\right)^{N_{vac}} - M_P. \tag{2.10}
\end{aligned}$$

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Requiring that that our universe with cosmological constant Λ is stable enough,

$$\Gamma_{(N_{vac})}^{\text{total}} < H = \frac{\sqrt{\Lambda}}{M_P} . \tag{2.11}$$

we obtain again an upper bound on Λ which now reads

$$\Lambda < \frac{24\pi^2 M_P^4}{\ln(N_{vac}/\ln 2)} . \tag{2.12}$$

This essentially agrees with the bound eq.(2.8) obtained before.

2.2 Decay of vacua in the landscape of string flux compactications

2.2.1 Vacuum decay for fixed background fluxes

As it is well know string compactifications lead to a huge number of lower dimensional ground states [12, 13, 14]. In particular the number N_{vac} of discrete vacua in the context of flux compactifications of type II orientifolds was estimated to be of order of $N_{vac} \sim 10^{500}$. Therefore a statistical analysis of flux vacua was suggested in [14, 15]. Wrapping in addition D-branes around cycles of the underlying (Calabi-Yau) spaces in order to derive the Standard Model of particle physics increases this number even further. Therefore intersecting brane models and the likelihood to derive the Standard Model were also investigated in a statistical manner [16, 17, 18, 19, 20, 21]. Here we want to discuss some constraints on the landscape of type II compactifications with p-form fluxes and also possible non-perturbative effects like gaugino condensation and Euclidean instantons, as it was proposed first in the KKLT scenario [11].

We will discuss flux compactifications in the context of the effective supergravity action. In a general $\mathcal{N} = 1$ supergravity, the scalar potential V is a function of chiral superfields ϕ_i and takes the standard form

$$V = e^K \left(|D_i W|^2 - 3|W|^2 \right) + |D_a|^2 , \tag{2.13}$$

where D_a are the D-terms, and the F-terms are defined as

$$F_i = e^{K/2} D_i W = e^{K/2} \left(\partial_{\phi_i} W + W \partial_{\phi_i} K \right) \tag{2.14}$$

with W being the superpotential and K the Kähler potential.

Our aim is to find local minima of V. We must therefore impose

$$\partial V/\partial\phi_i|_{\phi_{\min}} = 0 \; \forall i \, . \tag{2.15}$$

Supersymmetric minima are obtained if all $F_i|_{\phi_{\min}} = D_a|_{\phi_{\min}} = 0$.

Let us neglect the possible contribution of D-terms to the scalar potential. In this case V is fully specified by the Kähler potential K and the superpotential W in eq.(2.14). The generic form of the superpotential in type II orientifold compactifications is of the form

$$W = W_{\text{flux}}(\phi) + W_{n.p.}(\phi) \, . \tag{2.16}$$

On a generic Calabi-Yau space the total number of (type IIB) flux vacua is estimated by the following equation [13, 22, 23]:

$$N_{SUSY} \simeq \frac{L^{2h^{2,1}+2}}{(2h^{2,1} + 2)!} \tag{2.17}$$

Here the Hodge number $h^{2,1}$ counts the number of complex structure moduli, and L is the orientifold charge of the system. Typical numbers for $h^{2,1}$ and L indeed lead to a huge number of supersymmetric flux vacua. This number counts all different 3-form flux combinations that lead to a supersymmetric ground state satisfying that lead to a solution of the supersymmetry equations with respect to W_{flux}:

$$D_\phi W_{\text{flux}} = 0 \tag{2.18}$$

Including the non-perturabtive superpotential $W_{n.p.}$ and also looking for non-supersymmetric local, i.e. metastable vacua will not change this number by a considerable amount, i.e. the total number N_{vac} of local string vacua is comparable to N_{SUSY}: $N_{vac} \simeq N_{SUSY}$. In particular eq.(2.17) means that the huge number of flux vacua originates from the big number of possibilities of choosing different flux vectors though the homology 3-cycles of the CY space.

Let us first consider transitions between vacua with fixed values for the fluxes, i.e. all vacua have the same flux quantum numbers. These transitions are due to gravitational non-perturbative effects, like e.g. the Coleman/De Luccia instantons in case of positive cosmological constants, as describes above. However, since the fluxes are quantized and hence take discrete values, there exist only a few transitions that are possible. Indeed, on a given moduli space of type IIB complex structure moduli $\phi^{2,1}$, Kähler moduli $\phi^{1,1}$ and including the dilaton τ, transitions between different vacua are only possible for fixed

background fluxes, and also for fixed non-perturbative effects. I.e. fixing the flux parameters and also the non-perturbative superpotential, the corresponding scalar potential has only a relatively small number of (local) minima, denoted by N^*, and in general one has that $N^* << N_{vac}$. This fact largely restricts the possible vacuum decay processes within the string flux landscape for fixed fluxes. Varying the moduli fields, only a small subset of vacua can be reached by vacuum tunneling and decay processes, along the lines described in section two. E.g. in type IIB flux compactifications supersymmetric solutions are characterized by imaginary self-dual fluxes G_3 [24], whereas nonsymmetric local minima of V allow for more general flux choices. The extremality conditions comprise $h^{2,1} + 1$ conditions for $h^{2,1} + 1$ complex variables. Therefore one expects that the degeneracy in the moduli space is in general totally lifted, and one obtains a discrete set of solutions for the moduli fields. Their number N^* depends on the prepotential $F(U)$ of the underlying Calabi-Yau manifold. As one can show the number N^* of solutions of eqs. (2.18) is essentially of order one. E.g. consider a GVW/TV superpotential [25, 26] of the form [27]

$$W_{\mathbb{IIB}} = (p + iqSU_1)(l_2 - il_1U_2 + in_1U_3 - n_2U_2U_3).$$ (2.19)

p, q, l_1, l_2, n_2, n_2 parametrize the flux quantum numbers that are constrined by the tadpole condition. For fixed flux quantum numbers there is a unique solution of the supersymmetry condition with zero vacuum energy:

$$SU_1 = -\frac{p}{q}, \quad U_2 = \sqrt{\frac{l_1 l_2}{n_1 n_2}}, \quad U_3 = \sqrt{\frac{l_2 n_1}{l_1 n_2}}.$$ (2.20)

How many other local (non-supersymetric) minima of V may exist besides the supersymmetric Minkowski or AdS_4 groundstates? The answer to this question in general depends on the details of the non-perturbative part of the superpotential and also on the up-lift procedure, e.g. by additional D-terms or non-supersymmetric contributions to the potential. In general, we expect that the total number N_{vac} of vacua of different possible flux choices is by far larger that the number N^* of local minima of the scalar potential with fixed fluxes. This can be seen as follows: In KKLT [11] the modification of the IIB flux superpotential by non-perturbative D-instantons or by gaugino condensation is of the following form:

$$W = W_0(U) + A(U)e^{-aT}.$$ (2.21)

The fluxes entirely enter in W_0, which can be treated in some approximation as a constant contribution to the superpotential. Each different choice for the fluxes leads to some specific W_0. However for given fluxes, i.e. given W_0, the number of local minima of the

scalar potential is low. If we vary all moduli parameters plus the dilaton field, we are moving in a moduli space $\mathcal{M}$ of (complex) dimension $dim(\mathcal{M}) = (h^{1,1} + h^{2,1} + 1)$. On general grounds, we expect that the number N^* of solutions of eq.(2.15) is at most of the order $dim(\mathcal{M})$. This is obviously smaller that the number N_{SUSY} given eq.(2.17). Finally uplifting the potential by a small amount, in oder to obtain a vacuum with small cosmological constant V_0, will not drastically change the number N^* of metastable vacua.

2.2.2 Vacuum decay due to stringy domain walls

In order to get transitions between vacua with different flux quantum numbers, one needs non-perturbative, gravitational configurations which are coupled to the flux background fields, and which interpolate between different flux vacua. These are given in term of BPS or nearly BPS domain walls (membranes) (for earlier work see e.g. [28, 29]) in four-dimensional space time that are coupled to the scalar moduli fields. The profile of the domain wall is such that it separates spatial regions with different flux quantum numbers from each other. For the case that the domain wall is interpolating between two supersymmetric vacua, the interpolating solutions is describing a BPS domain wall. Of course, eventually we are interested in the decay of a non-supersymmetric flux vacuum with positive cosmological constant (our vacuum) and broken space-time supersymmetry into another (supersymmetric) flux vacuum, which cam have either positive, zero or also negative cosmological constant (AdS_4) vacuum. The formation of an AdS_4 domain wall is particularly interesting, since AdS_4 are very common in the string landscape. In this case our universe would be decaying into a contracting space, which at first sight seems to be problematic. Nevertheless the corresponding transition amplitude from dS_4 to AdS_4 is expected to be non-vanishing, as it was discussed in [30].

To demonstrate a vacuum transition between string vacua with different fluxes, we discuss as a simple example we type IIA, AdS_4 flux vacua with all moduli fixed at finite values. The corresponding domain walls were recently constructed in [31], and they are microscopically composed of intersecting D-branes, NS 5-branes and possibly also by socalled Kaluza-Klein monopoles. Specifically, consider a flux superpotential of the form [32, 33, 34, 35, 31]

$$W_{\text{IIA}} = W_H + W_F \,. \tag{2.22}$$

The first term is due to the Neveu–Schwarz 3-form fluxes and depends on the dilaton S

and the type IIA complex-structure moduli U_m $(m = 1, \ldots, \tilde{h}^{2,1})$:

$$W_H(S, U) = \int_Y \Omega_c \wedge H_3 = i\tilde{a}_0 S + i\tilde{c}_m U_m \,, \qquad (2.23)$$

where in type IIA the 3-form Ω_c is defined by $\Omega_c = C_3 + i\mathrm{Re}(C\Omega)$. Second, we have the contribution from Ramond 0-, 2-, 4-, 6-form fluxes:

$$
\begin{aligned}
W_F(T) &= \int_Y e^{J_c} \wedge F^{\mathrm{R}} \\
&= \tilde{m}_0 \frac{1}{6} \int_Y (J_c \wedge J_c \wedge J_c) + \frac{1}{2} \int_Y \left(F_2^{\mathrm{R}} \wedge J_c \wedge J_c\right) + \int_Y F_4^{\mathrm{R}} \wedge J_c + \int_Y F_6^{\mathrm{R}} \\
&= i\tilde{m}_0 F_0(T) - \tilde{m}_i F_i(T) + i\tilde{e}_i T_i + \tilde{e}_0 \,.
\end{aligned}
\qquad (2.24)
$$

Here $F(T) := F_0(T)$ is the type IIA prepotential, which depends on the IIA Kähler moduli T_i $(i = 1, \ldots, \tilde{h}^{1,1})$ and $F_i(T) := \partial F_0 / \partial T_i$. We use the notation J_c for the complexified Kähler metric $J_c := B + iJ$. Assuming a simple (toroidal) cubic prepotential $F = T_1 T_2 T_3$, the superpotential has the generic form:

$$
\begin{aligned}
W_{\mathrm{IIA}} &= W_F + W_H = \tilde{m}_0 \int_Y (J \wedge J \wedge J) + \int_Y F_4^{\mathrm{R}} \wedge J + \int_Y \Omega_c \wedge H_3 \\
&= i\tilde{e}_i T_i + i\tilde{m}_0 T_1 T_2 T_3 + i\tilde{a}_0 S + i\tilde{c}_m U_m \,.
\end{aligned}
\qquad (2.25)
$$

With $K = -\log(S + \bar{S}) \prod_{i=1}^3 (T_i + \bar{T}_i) \prod_{i=1}^3 (U_i + \bar{U}_i)$, the equations (2.18) admit the following unique solution with all moduli stabilized:

$$|\gamma_i| T_i = \sqrt{\frac{5|\gamma_1 \gamma_2 \gamma_3|}{3\tilde{m}_0^2}}\,, \quad S = -\frac{2}{3\tilde{m}_0 \tilde{a}_0} \gamma_i T_i \,, \quad \tilde{c}_m U_m = -\frac{2}{3\tilde{m}_0} \gamma_i T_i \,, \quad \gamma_i = \tilde{m}_0 \tilde{e}_i \,. \quad (2.26)$$

This solution corresponds to supersymmetric AdS_4 vacuum with negative cosmological constant:

$$\Lambda_{AdS} = -3e^K |W|^2 = -\frac{3^7 \sqrt{\frac{3}{5}}}{100} \frac{|\tilde{a}_0 \tilde{c}_1 \tilde{c}_2 \tilde{c}_3| (|\tilde{m}_0 \tilde{e}_1 \tilde{e}_2 \tilde{e}_3|)^{5/2}}{(\tilde{e}_1 \tilde{e}_2 \tilde{e}_3)^4} M_P^4 \,. \qquad (2.27)$$

Now let us consider the corresponding the domain wall solution which interpolates between the above AdS_4 flux vacuum and flat Minkowski space-time with vanishing fluxes. As discussed in [31] it is given in terms of intersting D4,- D8- and NS 5-branes. In addition one also needs orientifold 6-planes (O6) in order to cancel the induces D6-brane charge from the fluxes. The complete form of the 10-dimensional metric as well as the profiles of the scalar fields can be found in [31]. The four dimensional part of the metric is such of an interpolating domain wall, where the intersecting branes are smeared in the direction

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transveral to the domain wall. Specifically, this 4-dimensional part of the metric can be written as

$$ds^2 = a(r)^2(-dt^2 + dx^2 + dy^2) + dr^2 . \tag{2.28}$$

For $r \to 0$ this metric approaches the metric of AdS_4, and the scalar fields are fixed to the values determined by the non-vanishing fluxes, as given in eq.(2.26). For $r \to \infty$, the function $a(r)$ becomes a constant, and the eq.(2.28) become the metric of flat Minkowski space.

The tension σ of the domain wall can be computed by introducing a central function $Z(r)$ which is defines as

$$Z(r) = \frac{a'(r)}{a(r)} . \tag{2.29}$$

By comparison with the exact metric of [31] one obtains

$$Z(r)|_{r=0} = e^{K/2}|W| , \quad \Lambda_{AdS} = -3|Z(r)|^2_{r=0} . \tag{2.30}$$

The (membrane) tension σ of the domain wall is then given by the following expression:

$$\sigma \simeq (|Z|_{r=\infty} - |Z|_{r=0}) . \tag{2.31}$$

Now let us determine the decay amplitude of the Minkowski vacuum with vanishing fluxes into the AdS_4 vacuum with non-vanishing fluxes. The decay of the Minkowski vacuum occurs due to the creation of the domain wall, which speews through space-time until the entire universe is in the new AdS_4 vacuum. This is similar but not completely equal to the creation of a bubble via the Coleman/De Luccia instanton. In fact in order to be realistic, one should break supersymmetry and uplift the Minkowski vacuum by a small amount to obtain a de Sitter vacuum which decays into the AdS_4 vacuum. Neglecting the problem of supersymmetry breaking and the uplift, the decay amplitude of the Minkowki (de Sitter) vacuum is then given by the following expression:[4]

$$\Gamma \simeq M_P \exp\left(-\frac{8\pi^2 M_P^4 C}{\sigma^2}\right) = M_P \exp\left(\frac{24\pi^2 M_P^4 C}{\Lambda_{AdS}}\right) . \tag{2.32}$$

The constant C depends on the details of the domain wall solution.

As also discussed in [30], unlike the cases discussed in section two, the corresponding decay amplitude is independent of the de Sitter cosmological constant $\Lambda = V_0$, but only

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[4]The details of the derivation of this equation by computing the Euclidean action of the domain wall of ref. [31] coupled to the scalar fields will be given elsewhere [36].

depends on the value of Λ_{AdS}. In order to avoid too fast decay of our vacuum, $|\Lambda_{AdS}|$ must not be too large. E.g. if $|\Lambda_{AdS}| \simeq m_{3/2}^4$, the life-time of our universe is long enough. However AdS_4 vacua with $|V_1| \sim M_P^4$ create too much decay of our vacuum. Using the known expression for Λ_{AdS} in eq.(2.27), this constraint can be translated into the following restriction on the flux quantum numbers:

$$\frac{3^7 \sqrt{\frac{3}{5}}}{100} \frac{|\tilde{a}_0 \tilde{c}_1 \tilde{c}_2 \tilde{c}_3| (|\tilde{m}_0 \tilde{e}_1 \tilde{e}_2 \tilde{e}_3|)^{5/2}}{(\tilde{e}_1 \tilde{e}_2 \tilde{e}_3)^4} << 1 . \tag{2.33}$$

3   Black Hole Proof for de Sitter

Before studying applications of the bound on species (1.1) to the vacuum landscape, we wish to generalize the BH proof of the bound to the de Sitter and quasi de Sitter spaces. Let M be the mass of the species, and let H be the Hubble parameter in de Sitter. We wish to perform a thought experiment [1], in which number of species is absorbed by a BH, which then evaporates and releases them back. The key point is, that the BH can start emitting the species only after its Hawking temperature becomes comparable to their mass, and this fact implies (1.1). The necessary requirement for such an experiment is that the gravitational radius $r_g \equiv M_{BH}/M_P^2$ of the BH of the interest, must be less than the Hubble radius

$$r_g \ll H^{-1}. \tag{3.1}$$

We shall split the rest of the discussion into two parts, by imposing different constraints on the BH lifetime. This is dictated by the fact that for the validity of such an experiment, not only the size but the lifetime of the BH also matters. What is important, is that vacuum itself must be longer lived than the BH. This implies different constraints on the type of BH that we can use in our analysis for the vacua with different level of time-dependence.

3.1   Time-Dependent Vacua: Constraints from Short-Lived Black Holes

In the first case, let us require that not only the gravitational radius, but also the lifetime of the BH be less than the Hubble time. That is,

$$\tau_{BH} \ll H^{-1}. \tag{3.2}$$

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Notice, that the lifetime of a black hole depends on the number of species into which it can evaporate, and which in our case may be very large. For small black holes, $r_g \ll M^{-1}$, this correction can be very important, and must be taken into the account. On the other hand, for large BH $r_g \gg M^{-1}$, that mostly evaporate into few very light species (such as a graviton or a photon), the correction to the lifetime from N heavy states is unimportant, and for these BH the life-time is approximately given by

$$\tau_{BH} \sim r_g^3 M_P^2. \tag{3.3}$$

In practical applications, the requirement (3.2) will be relevant for the vacua that have a relatively short life-time, e.g., such as the slow-roll inflationary vacua, which can be regarded as stationary only for several Hubble times.

Notice, that since in any sensible (quasi) de Sitter state $H^{-1}M_P \gg 1$, the condition (3.2) outomatically implies (3.1). That is, a BH that evaporates in less than a Hubble time, is automatically small enough to fit within the Hubble horizon. Let us now prepare such a BH, by putting together n particles, all from different species. The maximal number of particles that we can add to a BH, without violating the requirement (3.2) (and automatically (3.1)), is limited by the following consideration.

In order to fit a particle into a BH, the typical momentum of the particle (that is, its characteristic inverse localization width) must be higher than r_g^{-1}. Indeed, even if a particle in question is massless, in order to throw it into a BH, we have to prepare a localized wave-packet of the size $\Delta X \lesssim r_g$. Such a wave-packet will have a characteristic momentum $\Delta P \gtrsim r_g^{-1}$. Thus, throwing a particle of the rest mass M into a BH, we automatically increase the mass of the latter at least by $\Delta M_{BH} \simeq \sqrt{M^2 + r_g^{-2}}$, and correspondingly, increase its horizon by

$$\Delta r_g \sim \frac{\sqrt{M^2 + r_g^{-2}}}{M_P^2}. \tag{3.4}$$

(Notice, that the converse is also true. When a black hole emits a particle, due to the thermal nature of Hawking radiation, the typical energy released is $\sim r_g^{-1}$, and decrease in the horizon is (3.4)). To find the BH mass as a function of number n of the "constitutent" particles, we must summ over all the increments. Approximating the sum by the integral, we get the following expression for the number of particles necessary for building a BH of a given mass M_{BH},

$$n(M_{BH}) \simeq \int_0^{M_{BH}} \frac{dm}{\sqrt{M^2 + M_P^4/m^2}} = \frac{1}{2M^2} \left(\sqrt{M^2 M_{BH}^2 + M_P^4} - M_P^2 \right). \tag{3.5}$$

The maximum number of particle species, $\bar{n}$, that can participate in our experiment, is set by the number of particles that is needed to grow the BH to a critical mass, M_C, with the lifetime becoming comparable to H^{-1}. That is, $\bar{n} \equiv n(M_C)$, where M_C saturates the bound (3.2). At this point, it is usefull to split the discussion into two parts, corresponding to the cases when $M \gg H$, and $M \lesssim H$.

3.1.1 Constraint on Heavy Species: $M \gg H$

This case requires a careful analysis, since the black hole lifetime, which is a function of its size r_g, undergoes an abrupt transition around the critical size $r_g \sim M^{-1}$. The reason is, that the black holes of size $r_g \lesssim M^{-1}$ have Hawking temperature $T_H \gtrsim M$, and can radiate all the constituent species. So the lifetime of a subcritical BH is

$$\tau_{BH}(r_g \lesssim M) \sim \frac{1}{n} \frac{M_{BH}^3}{M_P^4} . \tag{3.6}$$

Recall that our experiment is designed in such a way, that we are forming a minimal BH out of n particles belonging to *different* species. So due to conservation of species number, such a minimal BH can only radiate the n particles belonging to the input species, and not other energetically available $N - n$ species. In order to find number of species needed for building a BH of size $r_g \sim M^{-1}$, we just have to take $M_{BH} = M_P^2/M$ in eq(3.5). Ignoring the factors of order one, this gives

$$n(M_P^2/M) \sim \frac{M_P^2}{M^2}. \tag{3.7}$$

Plugging this into the eq(3.6), we get

$$\tau_{BH} \sim M^{-1}. \tag{3.8}$$

This result is already indicative. Equation (3.7) is compatible with the flat space bound, which is certainly applicable because the lifetime of a BH is $M^{-1} \ll H^{-1}$. Moreover, the fact that N cannot exceed M_p^2/M^2 can be anticipated from the fact that if it could, we could form a neutral BH of mass M^{-1}, with even less lifetime. This would indicate that such BHs cannot be treated as well defined states, in agreement with the result of [4].

So far, what we know is, that a BH of size $\sim M^{-1}$ can contain maximum M_P^2/M^2 units of the conserved species number. Now, let us try to build a bigger BH by putting in more species (as before, we keep adding particles from the different species!). Once the BH grows $r_g \gg M^{-1}$, there is sharp increase in its lifetime, since the emission of states

with mass M becomes exponentially suppressed by the Boltzmann factor. At this point, the BH can only evaporate into the small number of available massless species (such as the graviton), and the lifetime is given by (3.3).

Requiring that the resulting BH satisfies the lifetime constraint (3.2), that is,

$$r_g < (H^{-1}M_P)^{\frac{1}{3}} M_P^{-1}, \tag{3.9}$$

we find from (3.5) the corresponding maximal $\bar{n}$, by taking the integral up to $M_{BH} = (H^{-1}M_P)^{1/3}M_P$. Since, by default, the size of this BH is $\gg M^{-1}$, we have $M M_{BH} \gg M_P^2$ and the first term dominates in the last equation of (3.5). This gives

$$\bar{n} \sim \frac{M_P}{M}(H^{-1}M_P)^{\frac{1}{3}}. \tag{3.10}$$

Again, $\bar{n}$ sets the maximal number of particles that can participate in the experiment, without making BH unacceptably long lived. Now, if $N > \bar{n}$, the following constraint emeges. Using a subset of $\bar{n}$ species and peforming the thought experiment with the BH formation and evaporation, we arrive to the usual flat space constraint

$$\bar{n} \lesssim \frac{M_p^2}{M^2}. \tag{3.11}$$

From here, by taking into the account (3.10), we get the following bound on the mass of the species

$$M \lesssim \frac{M_P}{(H^{-1}M_P)^{\frac{1}{3}}}. \tag{3.12}$$

3.1.2 Constraint on Light Species: $M \lesssim H$

In such a case, even a BH as big as $\sim H^{-1}$, can have a lifetime $\sim H^{-1}$. So, for finding $\bar{n}$ in eq(3.5) the integration must be performed up to $M_{BH} = H^{-1}M_P^2$. Since $\frac{M_P^2}{M} \gg H \gg M$, this gives,

$$\bar{n} \sim (H^{-1}M_P)^2. \tag{3.13}$$

Checking for the lifetime, we get

$$\tau \lesssim \frac{1}{\bar{n}}H^{-3}M_P^2 \sim H^{-1}, \tag{3.14}$$

which confirms the legitimacy of the derivation. Then again, because by default $\bar{n}$ has to satisfy the bound (3.11), we get

$$(H^{-1}M_P)^2 < M_P^2/M^2, \tag{3.15}$$

which is automatically compatible with the original assumption that $H > M$. What remains is to be seen that $N \leq \bar{n}$. This follows from the Gibbons-Hawking temperature constraint. Indeed, because the de Sitter space is a thermal bath with effective temperature $T_{GH} \sim H$, the contribution to the energy density from N species with masses $M < H$ would be

$$\rho_{species} \sim N\, H^4. \tag{3.16}$$

This contribution cannot exceed the energy density of the de Sitter vacuum, which puts the upper bound on the number of species lighter than H, to be M_P^2/H^2. Notice, that for species that are lighter than H, this is a more stringent bound, than the flat space one.

3.2 Classically Stable Vacua: Relaxing the Longevity Constraint

In the analysis of the previous section, we have deliberately limited ourselves by considering BH that are sufficiently short-lived. This requirement is certainly justified for the time dependent vacua, which can only be regarded as stationary de Sitter on the time-scales of few Hubble. Most of the slow-roll inflationary vacua fall in this category.

On the other hand, the vacua that correspond to the classically-stable minima of the landscape, are exponentially long lived. For such vacua, the requirement (3.2), demanding that the BH evaporation time to be less than the Hubble time, is unnecessarily stringent. Indeed, we can have a hypothetical observer orbiting around a BH on a stationary orbit for much longer than the Hubble time. What is important in such a case, is that the lifetime of the BH is longer than the lifetime of the vacuum τ_{vac}. If latter is the case, we can relax the requirement (3.2) and only demand (3.1). It is again useful to split the discussion in two cases, corresponding to the mass of the species being heavier or lighter than the Hubble.

3.2.1 Constraint on Heavy Species: $M \gg H$

Again we first have to find the number of available species $\bar{n}$, which can participate in the BH formation and evaporation experiment that are compatible with the constraint (3.1). This can be found by integrating (3.5) up to the mass of the Hubble size BH, which has

a mass $M_{BH} \sim H^{-1}M_P^2$. This gives

$$\bar{n} \sim \frac{M_P^2}{MH}. \tag{3.17}$$

An alternative way of finding the maximal number of heavy species of mass $M \gg H$, that can participate in the experiment, is by estimating of how many such particles can fit within the de Sitter horizon before turning the Hubble volume into a BH,

$$\frac{\bar{n}M}{M_P^2} \sim H^{-1} \quad \rightarrow \quad \bar{n} \sim \frac{M_P^2}{HM}. \tag{3.18}$$

Because $M \gg H$, the above number is much larger than the flat space bound on the number of species. Thus, in this case, de Sitter is essentially not limiting the number of species that one could use in BH formation, and the flat space BH bound remains. Thus we have,

$$N \lesssim \frac{M_P^2}{M^2}, \tag{3.19}$$

just as in flat space. This makes perfect sense. Indeed, in an eternal de Sitter space, sub-horizon BHs formed by the heavy particles, evaporate just as in the flat space.

3.2.2 Constraint on Light Species: $M \lesssim H$

In this case expression for $\bar{n}$ (again the maximal number of particles that can be used in experiment without conflicting with (3.1)) changes to

$$\bar{n} = (H^{-1}M_P)^2. \tag{3.20}$$

Let us find out, what is the constraint on N in such a case. Let us first show, that we cannot have $N > \bar{n}$ due to Gibbons-Hawking temperature argument. Because

$$M \lesssim H, \tag{3.21}$$

in the de Sitter space all the species contribute to Gibbons-Hawking radiation. Each species with mass $< H$, will contribute into the thermal energy a factor $\sim H^4$, which for $N > \bar{n}$ would exceed the energy in de Sitter space. This is impossible. Thus, we arrive to the conclusion that $\bar{n}$ is the bound on N. Thus,

$$N \lesssim \frac{M_P^2}{H^2}. \tag{3.22}$$

Again, this result agrees with the general intuition, since in the presence of sub-Hubble mass species, the de Sitter horizon strongly limits the size of the BH that in the flat space

would evaporate into the light species. Thus, the key point is that for the light species $M \ll H$, the bound is cut-off by the Gibbons-Hawking temperature argument, which is more stringent than the flat space bound of [1].

We shall now apply this consideration to different inflationary scenarios.

4 Application for the Landscape

4.1 Stationary SUSY-Breaking de Sitter Vacua

In this section, we shall apply our consideration to the vacua that are classically stable, and thus have an exponentially long life-time.

Consider a nearly Minkowski vacuum in which gravitino mass is $m_{3/2}$. In the standard picture our MSSM vacuum is such. In this vacuum there are moduli that are getting masses from the SUSY-breaking dynamics, and their masses are $\sim m_{3/2}$. These moduli parameterize the would be flat directions, that are lifted by SUSY-breaking. When we move along the lifted flat directions, many particles become massive. Let such modulus be ϕ. For example, ϕ can be one of the MSSM flat directions. In F-term type supersymmetry breaking, the potential for moduli is generated through the Kähler couplings to the SUSY-breaking F-terms and has a form

$$V(\phi) = m_{3/2}^2 M_P^2 \, \mathcal{V}(\frac{\phi}{M_P}). \tag{4.1}$$

Usually, is it assumed that the function $\mathcal{V}(\phi/M_P)$ can have many new minima at values $\phi \sim M_P$. However, the BH bound derived in the previous section can restrict such possibilities.

To see this, imagine that indeed there is a new minimum at $\phi \sim M_P$. Of course, typically this minimum will not be Minkowski and will have a vacuum energy of order the SUSY-breaking scale $V_0 \sim m_{3/2}^2 M_P^2$. The question is, what is the restriction on the number of species of mass M in such a vaccum.

In this section we will be interested in classically-stable vacua, which can only decay through the tunneling process and thus, have an exponentially-long lifetime τ_{vac}. We shall show, that BH consideration of the previous section can provide a restriction on this lifetime in terms of number of species N and their mass M. For definiteness, we shall discuss vacua with $M \gg m_{3/2}$.

In order to see this, let us first assume that the vacuum in question can be arbitrarily long lived. In particular, τ_{vac} can be much longer than the lifetime of a minimal BH satisfying the constraint (3.1). That is

$$\tau_{vac} \gg \tau_{BH} \sim \frac{(\bar{n}M)^3}{M_P^4} \sim H^{-3}M_P^2, \tag{4.2}$$

where in the last expression we have taken in to the account (3.17). Then, as shown in the previous section the BH proof of the bound, $NM^2 < M_P^2$, will go through. The requirement that the gravitational radius r_g of a minimal BH incorporating all the species, is less than the curvature radius of the vacuum (H^{-1}), applied to SUSY-breaking vacua, takes the form

$$r_g \lesssim H^{-1} \quad \rightarrow \quad NM \lesssim M_P^2/m_{3/2}, \tag{4.3}$$

where we have used the fact that the mass of a minimal BH (containing all the species) is $M_{BH} \sim NM$. Because (4.3) implies

$$N \lesssim \frac{M_P^2}{M^2}\frac{M}{m_{3/2}}, \tag{4.4}$$

the flat space BH bound

$$N \lesssim \frac{M_P^2}{M^2} \tag{4.5}$$

is automatically valid even in the curved vacua (with $V_0 \sim m_{3/2}^2 M_P^2$), as long as, $M > m_{3/2}$.

Now it is obvious that the above result puts a severe restriction on all the vacua, that are obtained my modular deformation from the Minkowski vacuum in which supersymmetry breaking scale is hierarchically small. For instance, on the deformations of the standard MSSM vacuum in which the hierarchy problem is solved by the low energy SUSY-breaking. An immediate implication is that there cannot be the metastable vacua in which MSSM flat directions have $\gtrsim M_P$ VEVs, since such vacua would automatically fall within the conditions of the BH proof, and in the same time there many species will get masses $M \sim M_P$, in contradiction with this bound. The same is true for the deformations of the vacua with GUT symmetry breaking, and for many other cases.

What happens if the bound is not satisfied, for example, what if there are too many massive particles? Then, by consistency, theory has to respond by decreasing the lifetime of the vacuum, in such a way that (4.2) is no longer valid. That is, a large number of species must destabilize the vacuum! In such a case the vacuum in consideration becomes short lived or even classically unstable, and the argument has to be reconsidered. We shall discuss such a situation in the next section.

4.2 Constraint on the Slow-roll Inflationary States

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The black hole bound on species (1.1) can be extended not just to the (meta) stable vacua, but also to time dependent "vacua", with slowly changing values of the parameters. The important examples from this class of vacua are the inflationary slow-roll backgrounds. We shall now apply the BH bound to such states.

Consider a slow roll inflation driven by a single inflaton field ϕ. The equation for the spatially-homogeneous time-dependent field is,

$$\ddot{\phi} + 3H\,\dot{\phi} + V(\phi)' = 0\,, \tag{4.6}$$

where, prime stands for the derivative with respect to ϕ. The main idea of the slow roll inflation is, that for certain values of ϕ, the potential $V(\phi)$ is sufficiently flat, so that the friction term dominates and this allows ϕ to roll slowly. The energy density is then dominated by the slowly-changing potential energy. The Hubble parameter is approximately given by $H^2 \simeq V(\phi)/3M_P^2$, and can be regarded as constant on the time scales $\sim H^{-1}$. Obviously, the inflationary region of the potential must be away from todays minimum with almost zero vacuum energy. In any inflationary scenario the value of the inflaton field during inflation is very different from its todays expectation value ϕ_0 corresponding to the minimum of $V(\phi)$, which without loss of generality we can put at $\phi_0 = 0$.

Soon after the end of the inflationary period, inflaton oscillates about its true minimum ϕ_0, and reheats the Universe. For this to happen, inflaton should necessarily interact with the standard model particles and possibly with the other fields. Let us consider an inflaton coupled to N species, with masses M_j. For the efficient reheating, the masses of the the particles about the minimum ϕ_0, must be less than the inflaton mass about the same minimum. That is, $M_j \ll V''(\phi_0)$. Due to coupling to the inflaton field, the masses of species are functions of its expectation value, $M_j(\phi)$, and it is very common that these masses change substanctialy during inflation. The key point that we are willing to address now, is that the masses of these species are subject to the BH bound, and give useful restriction on the inflationary trajectory. Thus, knowing the couplings of the inflaton in *our* vacuum, one can get an non-trivial information about the much remote inflationary vacua of the same theory.

For simplicity, we shall assume the universality of the species masses $M_j(\phi) = M(\phi)$.

During the slow-roll inflation, Universe is in a quasi-de-Sitter state, in which the inflationary Hubble parameter sets the size of the causally-connected event horizon H^{-1}. However, the difference from the stationary de Sitter vacua, is that in realistic inflationary scenarios the slow roll phase (in any given region) is not exponentially long lived, and lasts for several Hubble times. So H^{-1} sets the time scale on which parameters can be regarded as constant.

Thus, a hypothetical observer located within a given causally-connected inflationary patch can perform a sensible experiment with BH formation and evaporation, as long as the gravitational radius r_g and the BH lifetime τ obey the bounds (3.1) and (3.2). In such a case, the considerations of section 1.1 can be directly applied, and we arrive to the bound,

$$M(\phi) \; < \; \frac{M_P}{(H^{-1}(\phi)M_P)^{\frac{1}{3}}}. \tag{4.7}$$

All the information that this bound implies for a given inflationary scenario, is encoded in the functions $M(\phi)$ and $H(\phi)$. We shall now illustrate this on some well known examples.

4.3 Chaotic Inflation

Let us consider the example of Linde's chaotic inflation [37]. This is based on a single scalar field with a mass m and no sef-coupling

$$V(\phi) \; = \; \frac{1}{2}m^2\phi^2 \; + \; g\phi\bar{\psi}_j\psi_j \,. \tag{4.8}$$

The last term describes the coupling to N-species, which for definiteness we assume to be fermions, and g is the interaction constant. As said above, the coupling of the inflaton to the species is crucial for the reheating.

The above theory has a Minkowski vacuum, in which $\phi = 0$ and all the species are massless. Due to the latter fact, in this vacuum the BH bound on the number and mass of the species is satisfied. However, as we shall see, the same bound, puts non-trivial restriction on the inflationary epoch, since during inflation $\phi \neq 0$ and species are massive.

Ignoring for a moment the coupling to the species, the logic in the standard Chaotic inflationary scenario goes as follows. The expectation value of the field ϕ can be arbitrarily large, as long as the energy density remains sub-Planckian, that is

$$m^2\phi^2 \; \ll \; M_P^4 \,. \tag{4.9}$$

The equation (4.6) then can be applied and takes the form

$$\ddot{\phi} + 3H\,\dot{\phi} + m^2\phi = 0\,, \tag{4.10}$$

where $H^2 = \frac{m^2\phi^2 + \dot{\phi}^2}{6M_P^2}$. As long as $H \gg m$, the friction dominates and ϕ rolls slowly. This implies (up to a factor of order one)

$$\phi \gg M_P, \tag{4.11}$$

which is compatible with (4.9) as long as $m \ll M_P$. If the above is satisfied, ϕ rolls slowly, and Universe undergoes the exponentially fast expansion. Let us now see how the coupling to the species restricts the above dynamics. During inflation the mass of the species is $M = g\phi$ and they are subject to the BH bound. To see what this bound implies we can simply insert the current values of $M(\phi)$ and $V(\phi)$ in (4.7), and we get

$$g\phi \lesssim M_P \left(\frac{m\phi}{M_P^2}\right)^{\frac{1}{3}}. \tag{4.12}$$

Non-triviality of the above constraint is obvious. For example, the standard argument assumes that inflation could take place for arbitrary $m \ll M_P$, and from arbitrarily large values of ϕ satisfying (4.9), irrespective to the number of species to which inflaton is coupled. The above expression tells us that in the presence of species, this is only possible, provided, $g \lesssim (M_P/\phi)^{2/3}(m/M_P)^{1/3}$.

For the practical reasons of solving the flatness and the horizon problems, in the standard Chaotic scenario, last 60 e-foldings happen for $\phi \lesssim 10M_P$, whereas from density perturbation we have $m \sim 10^{12}\text{GeV}$ or so. This implies, $g < 10^{-3}$. This constraint can be easily accommodated by the adjustment of couplings, however it is remarkable that no fine tuning can make $g \sim 1$ consistent.

4.4   Hybrid Inflationary Vacua

The essence of the hybrid inflation [38] is that inflationary energy density is not dominated by the potential of the slowly-rolling inflaton field ϕ, but rather by a false vacuum energy of other scalar fields, χ_j. These fields are trapped in a temporary minimum, created due to large positive mass^2-s, which they acquire from the coupling to the inflaton field. The slowly rolling inflaton then acts as a clock, which at some critical point triggers the transition that liberates the trapped fields, and converts their false vacuum energy into radiation. However, usually Inflation ends before this transition, because of breakdown

of the slow-roll. Thus, in hybrid inflation, the presence of fields with inflaton-dependent masses is essential not only for the reheating, but for the inflation itself.

The simplest prototype model realizing this idea is

$$V = \lambda^2 \phi^2 \chi_j^2 + \left(\frac{g}{2}\chi_j^2 - \mu^2\right)^2 , \tag{4.13}$$

where λ and g are constants. Then, for $|\phi| > \phi_t \equiv \mu\sqrt{\frac{g}{\lambda^2}}$, the effective potential for χ_j is minimized at $\chi_j = 0$, and the false vacuum energy density is a ϕ-independent constant, μ^4. Thus, in the classical treatment of the problem, starting at arbitrary initial value $\phi \gg \phi_t$ and with zero initial velocity, ϕ would experience zero driving force and system would inflate forever. One could slightly lift this flat direction by adding an appropriate self interaction potential for ϕ (e.g., such as a positive mass term $m^2\phi^2$) which would drive ϕ towards the small values. In such a picture inflation ends abruptly after ϕ drops to its critical value ϕ_t, for which χ_j becomes tachionic, and system rapidly relaxes into the true vacuum. However, the above story is only true classically, and quantum mechanical corrections are very important and always generate potential for ϕ [40, 41]. Because of to these corrections, typically, inflation ends way before the phase transition, due to breakdown of the slow-roll. Existence of supersymmetry cannot change the latter fact, however, supersymmetry does make the corrections to the potential finite and predictive.

The simple supersymmetric realizations of the hybrid inflation idea have been suggested in form of F-term [39, 40] and D-term [41, 42] inflationary models. As a result of supersymmetry, in F-term inflation $\lambda = g$. As it was shown in [40] and [41], due to renormalization of the Kähler function via χ_j loops, the non-trivial inflaton potential is inevitably generated, which for $\phi \gg \phi_t$ has the following form,

$$V(\phi) \simeq \mu^4 \left[1 + \frac{Ng^2}{16\pi^2}\ln\frac{g|\phi|}{Q}\right] , \tag{4.14}$$

where, Q is the renormalization scale. Notice, that this potential cannot be fine tuned away by addition of some local counter terms. The condition of the slow roll is that $V'' \ll H^2$, implying that

$$N g^2 \ll \frac{\phi^2}{M_P^2} . \tag{4.15}$$

Because of the logarithmic nature, the slope flattens out for large ϕ. However, even if one tries to ignore any other correction to the potential, nevertheless, the slow-roll condition will eventually run in conflict with the black hole bound, which implies that

$$N g^2 \lesssim \frac{M_P^2}{\phi^2} . \tag{4.16}$$

This fact indicates, that even if the theory is in seemingly-valid perturbative regime (that is, $\frac{Ng^2}{16\pi^2}\ln\frac{g\phi}{Q} \ll 1$), nevertheless, the perturbative corrections to the Kähler cannot be the whole story, and theory has to prevent growth of ϕ, by consistency with the black hole physics.

We wish to point out one subtle difference between the F-term and D-term inflationary scenarios. In case of F-term inflation, χ_j fields need not transform under any long range (un-Higgsed) gauge symmetry. However, in case of the D-term inflation story is more involved, because the mass parameter μ^2 comes from the Fayet-Illiopoulos term ξ of an $U(1)$ vector supermultiplet. In the globally supersymmetric limit, the potential has the form [41]

$$V = \lambda^2 |\phi|^2 \left(|\chi_j|^2 + |\bar{\chi}_j|^2\right) + \frac{g^2}{2}\left(|\chi_j|^2 - |\bar{\chi}_j|^2 - \xi^2\right)^2, \qquad (4.17)$$

where, χ and $\bar{\chi}$ carry opposite charges, which we take equal to $+1$ and -1 respectively. The mass of the $U(1)$ gauge field (call it W_μ) therefore vanishes above the critical point $|\phi|^2 > \phi_t^2 \equiv \frac{g^2}{\lambda^2}\xi$. However, this is an artifact of the global supersymmetry.

The most important effect of supergravity corrections to this picture is that $U(1)$ becomes a gauged R-symmetry [43], and the charges experience a shift of order ξ/M_p^2. This can be seen from the expression for the covariant derivative on the gravitino (we use conventions of [44], see details there)

$$\mathcal{D}_{[\mu}\psi_{\nu]} = \left(\partial_{[\mu} + \frac{1}{4}\omega_{[\mu}{}^{ab}(e)\gamma_{ab} + \frac{1}{2}A_{[\mu}\gamma_5\right)\psi_{\nu]}, \qquad (4.18)$$

where $\omega_\mu^{ab}(e)$ is the spin connection, and the $U(1)$-connection A_μ is given by

$$A_\mu = \frac{1}{2}\left[(\partial_i\mathcal{K})\hat{\partial}_\mu z^i - (\partial^i\mathcal{K})\hat{\partial}_\mu z_i\right] + \frac{g\xi}{M_P^2}W_\mu, \qquad (4.19)$$

where

$$\hat{\partial}_\mu z_i = \partial_\mu z_i - W_\mu\,\eta_i(z). \qquad (4.20)$$

Here, $\mathcal{K}$ is the Kähler function, and sum runs over all the chiral superfields z_i and $\eta_i(z)$ are the holomorphic functions that set the $U(1)$ transformations of all chiral superfields in the superconformal action,

$$\delta z_i = \eta_i(z)\alpha(x). \qquad (4.21)$$

When the Kähler potential is $U(1)$-invariant, as is the case in the simple model above, the $U(1)$ gauge transformation of the gravitino gauge-connection A_μ takes a universal form:

$$\delta A_\mu = \frac{g\xi}{M_P^2}\delta W_\mu = \frac{g\xi}{M_P^2}\partial_\mu\alpha(x), \qquad (4.22)$$

which means that gravitino acquires an $U(1)$-charge, and thus $U(1)$ becomes an R-symmetry.

Because of this charge shift, it is not at all guaranteed that $U(1)$ will stay un-Higgsed even though χ_j VEVs vanish. The characteristic mass of the $U(1)$ photon is at least as large as the Hubble parameter. As we shall discuss, this is exactly what happens in D-brane inflation.

4.5   Brane Inflationary Vacua

A possible mechanism for the inflation in string theory, is brane inflation [45, 46, 47, 48, 49, 50]. In this picture the role of the inflaton field ϕ is played by the brane-separation field. A simplifying but crucial assumption of the original brane inflation model, is that compactification moduli are all fixed, with the masses being at least of order of the inflationary Hubble parameter, so that branes can be considered to be moving in a fixed external geometry, weakly affected by the brane motion. In the same time, the $4d$ Hubble volume must be larger than the size of the compact extra dimensions. These conditions allow us to apply the power of the effective four-dimensional supergravity reasoning.

Below we shall focus on the case of $D-brane$ inflation, based on the motion and subsequent annihilation of branes an anti-branes. In [47], it was shown that this picture from the four-dimensional perspective can be understood as the hybrid inflation, in which ϕ is a brane distance field, and role of χ is played by the open string tachion.

An interesting evidence, indicating that D-brane inflation as seen from the $4d$ super-gravity perspective is of the D-term type, emerged later (see [51, 44]). This connection allows us to apply the above-derived black hole constraints to brane inflation both from $4d$ supegravity as well as from $10d$ string theory point of view.

In this picture, the supersymmetry breaking by a non BPS brane-anti-brane system corresponds to the spontaneous supersymmetry breaking via FI D-term.

When branes are far apart, there is a light field ϕ, corresponding to their relative motion. This mode is a combination of the lowest lying scalar modes of the open strings that are attached to a brane or anti-brane only. We are interested in the combination that corresponds to the relative radial motion of branes.

$$\phi = M_s^2 r \,, \tag{4.23}$$

where M_s is the string scale.

In the simplest case of a single brane-anti-brane pair, we have the two gauged $U(1)$-symmetries. One of these two provides a non-vanishing D-term. The tachyon (χ) is an open string state that connects the brane and the anti-brane. The mass of this stretched open string is $M_s^2 r$. In $4d$ language, the tachyon as well as other open string states get mass from the coupling to ϕ.

The energy of the system is given by the D-term energy, which is constant at the tree-level, but not at one-loop level. At one-loop level the gauge coupling depends on ϕ. g^2 gets renormalized, because of the loops of the heavy $U(1)$-charged states, with ϕ-dependent masses. For instance, there are one-loop contributions from the χ and $\bar{\chi}$ loops. More precisely there is a renormalization of g^2 due to one-loop open string diagram, which are stretched between the brane and anti-brane. Since the mass of these strings depend on ϕ, so does the renormalized D-term energy

$$V_D = \frac{g^2(\phi)}{2} D^2 = \frac{g_0^2}{2} \left(1 + g_0^2 f(\phi)\right) \xi^2 \,, \tag{4.24}$$

where g_0^2 is the tree-level gauge coupling, and $f(\phi)$ is the renormalization function. For example, for $D_3 - D_7$ system [52] at the intermediate distances ($M_s^{-1} \ll r \ll R$, where R is the size of two transverse extra dimensions), this takes the form (4.14).

We shall now see, why at least in the simplest D-brane setup, the $U(1)$ symmetry must be Higgsed throughout the inflation.

Let us again think about the process of $\mathrm{D}_{3+q} - \bar{\mathrm{D}}_{3+q}$ driven inflation, with the subsequent brane annihilation. We assume that q dimensions are wrapped on a compact cycle, and relative motion takes place in $6 - q$ remaining transverse dimensions.

The low energy gauge symmetry group is $U(1) \times U(1)$, one linear superposition of which is Higgsed by the tachyon VEV. The crucial point is, that this Higgsed $U(1)$ gauge field is precisely the combination of the original $U(1)$-s that carries a non-zero RR-charge (the other combination is neutral). The corresponding gauge field strength ($F_{(2)}$) has a coupling to the closed string RR $2 + q$-form ($C_{(2+q)}$) via the WZ terms,

$$\int_{3+1+q} F_{(2)} \wedge C_{(2+q)} \,, \tag{4.25}$$

where, since we are interested in the effective $4d$ supergravity description, we have to integrate over extra q-coordinates, and only keep the $4d$ zero mode component of the RR field. This then becomes an effective 2-form, $C_{(2)}$.

The connection with the $4d$ supergravity D-term language, is made by a dual description of the $C_{(2)}$-form in terms of an axion (a),

$$d\mathrm{C}_{(2)} \rightarrow *d\mathrm{a}\,, \tag{4.26}$$

where star denotes a $4d$ Hodge-dual. Under this duality transformation we have to replace

$$(d\mathrm{C}_{(2)})^2 \; + \; \frac{\xi}{M_P^2} F_{(2)} \wedge \mathrm{C}_{(2)} \quad \rightarrow \quad M_P^2(d\mathrm{a} \, - \, \mathrm{g}\mathrm{Q}_\mathrm{a}\mathrm{W})^2\,, \tag{4.27}$$

where $Q_a = \frac{\xi}{M_P^2}$ is the axion charge under $U(1)$. As it should, this charge vanishes as the compactification volume goes to infinity, and $4d$ supergravity approaches the rigid limit[5]. We thus see that the $U(1)$ gauge field (W_μ) acquires a mass $m_W^2 \gtrsim \xi^2/M_P^2$.

We are now ready to discuss applicability of our BH thought experiment to the above D-brane system. Since the role of the species χ_j, that are getting mass from the inflaton field, is played by the stretched open strings, the first condition for the applicability of the BH bound is, that these strings should fit at least within the Hubble size black hole. This is automatically the case, since by the validity of the brane inflation, the effective $4d$ Hubble volume must be much larger than the size of the compactified dimensions. Since the length of the stretched strings cannot exceed the latter size, they automatically fit within the black hole horizon.

The second issue is the possible interference of the $U(1)$ "hair" of the open string tachion with the black hole formation and evaporation process. Again, as we have seen, the black holes of interest have size of order Hubble, which is comparable to the Compton wavelength of the $U(1)$-photon. On the other hand, stretched strings are heavy, so the lifetime of such a black hole is many Hubble times. Typical time scale for a black hole to loose a photon hair is the Hubble time (because of photon mass), after this time, black holes should evaporate as normal hairless black holes. So again, at least to leading order, the massive $U(1)$ photon should not interfere with our arguments.

We should stress, however, that because the photon mass is roughly the same order as the curvature scale, more careful analysis would be very useful. This will not be attempted here.

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[5]The above value of the axionic charge, reproduces the correct RR charge of the D_1-string, and also has a correct scaling for the anomaly cancellation[44].

Acknowledgments:

Note Added:

Before submitting this paper, ref. [53] appeared, which discusses some constraints on ϕ^4-inflation in the presence of species from a different perspective.

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